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# Graph-based dependency grammar

Syntactic analysis (5LN455)

2023

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Partially based on slides from Marco Kuhlmann



# Overview

- Dependency grammar and projectivity
- Arc-factored dependency parsing
  - Collins' algorithm
  - Eisner's algorithm
- Evaluation of dependency parsers
- Transition-based dependency parsing
  - The arc-standard algorithm
- Advanced dependency parsing

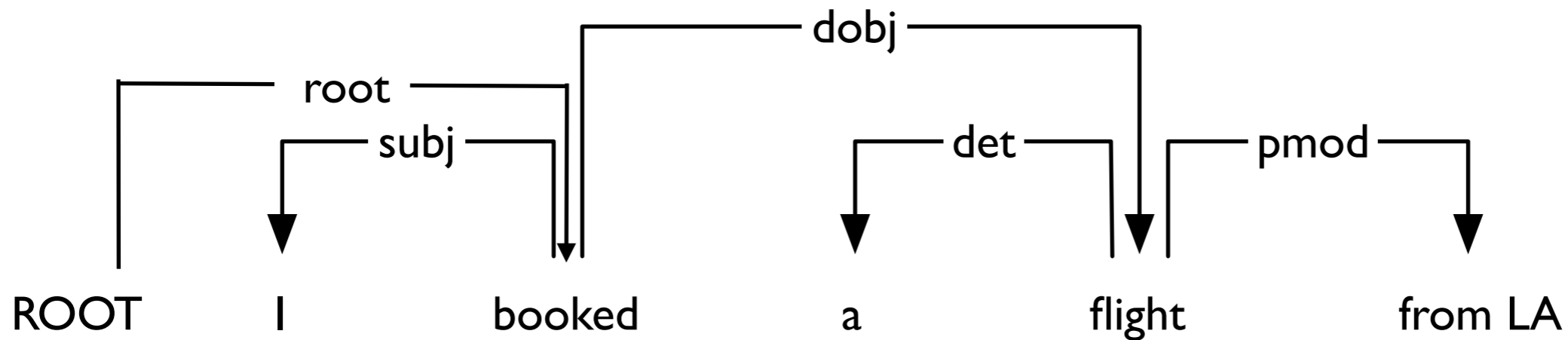


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# Dependency grammar



# Dependency trees



- In an arc  $h \rightarrow d$ , the word  $h$  is called the **head**, and the word  $d$  is called the **dependent**.
- The arcs form a **rooted tree**.
- Each arc has a **label**,  $l$ , and an arc can be described as  $(h, d, l)$

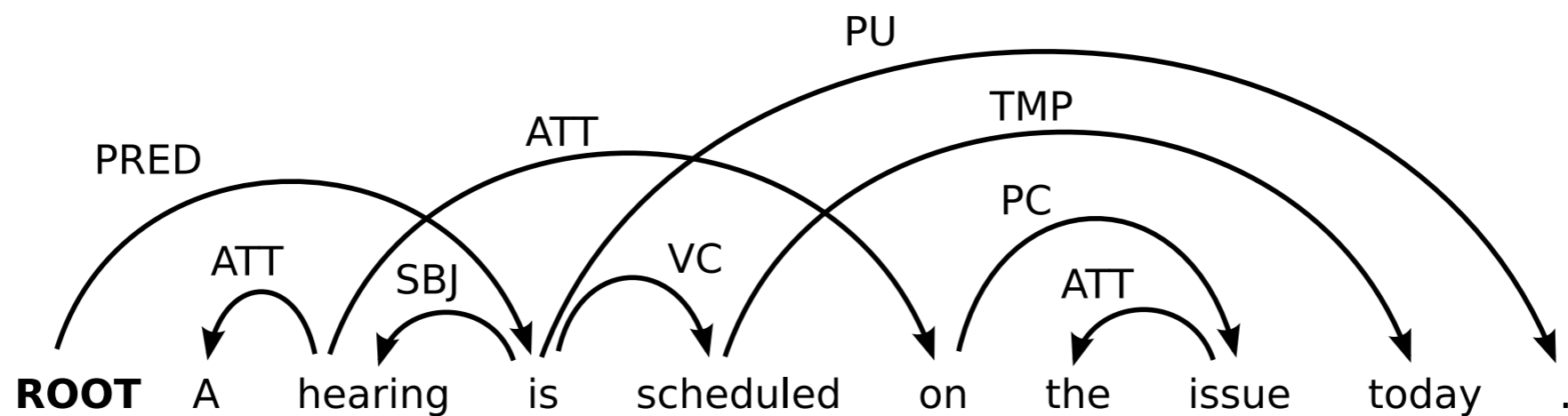
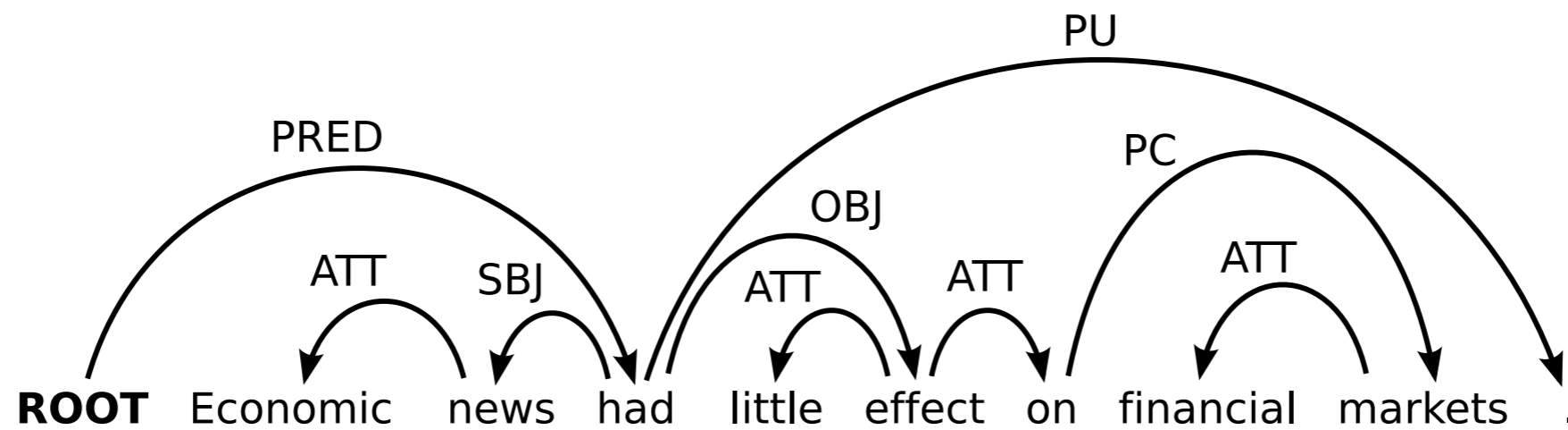


# Projectivity

- An important characteristic of dependency trees is projectivity
- A dependency tree is projective if:
  - For every arc in the tree, there is a directed path from the head of the arc to all words occurring between the head and the dependent (that is, the arc  $(i,l,j)$  implies that  $i \rightarrow^* k$  for every  $k$  such that  $\min(i, j) < k < \max(i, j)$ )



# Projective and non-projective trees





# Projectivity and dependency parsing

- Many dependency parsing algorithms can only handle projective trees
- Non-projective trees do occur in natural language
  - How often depends on the language (and treebank)
- In the course: in-depth discussion of projective algorithms, some discussion of non-projective algorithms



# Main parsing strategies

- Graph-based dependency parsing:
  - Scores the dependency graph (tree)
- Transition-based dependency parsing:
  - Scores a sequence of transitions
- There are also grammar-based methods, which we will not discuss (not commonly used)





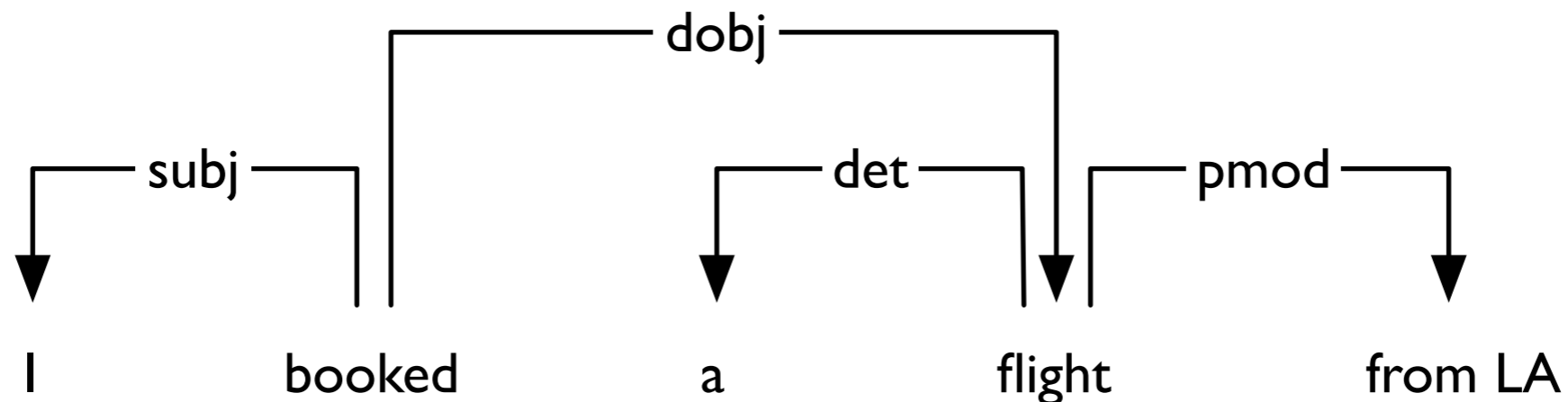
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# Arc-factored dependency parsing



# Ambiguity

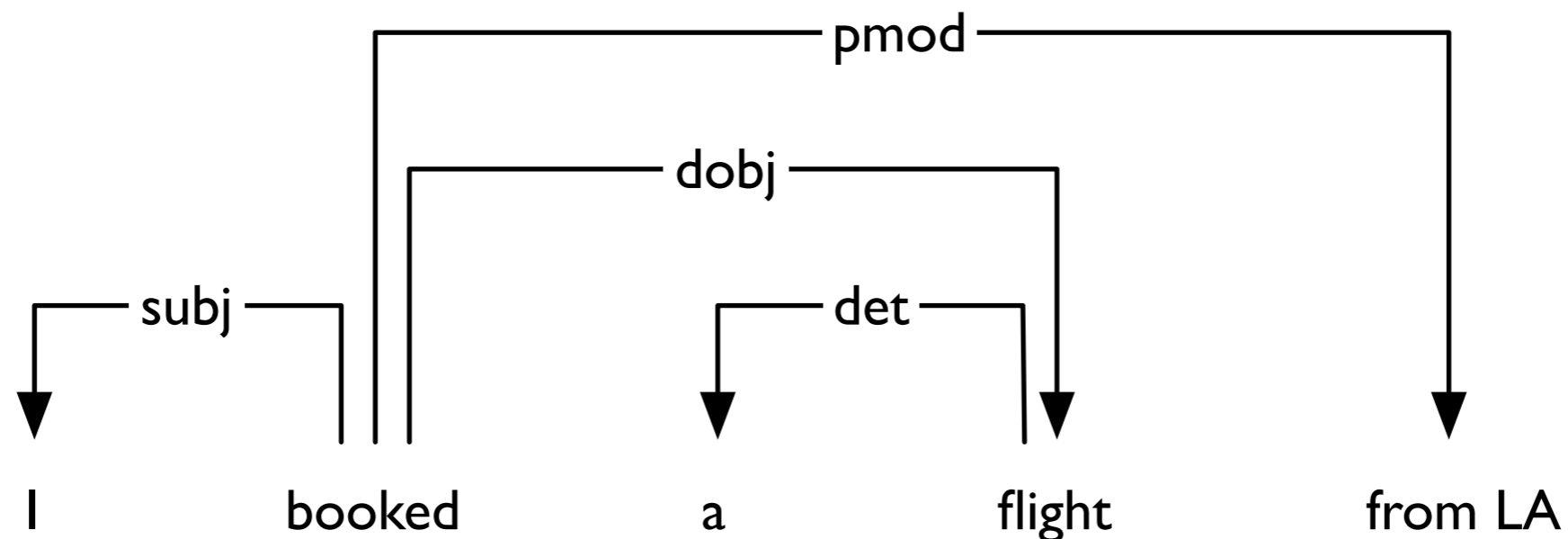
Just like phrase structure parsing,  
dependency parsing has to deal with ambiguity.





# Ambiguity

Just like phrase structure parsing,  
dependency parsing has to deal with ambiguity.





# Disambiguation

- We need to **disambiguate** between alternative analyses.
- We develop mechanisms for scoring dependency trees, and disambiguate by choosing a dependency tree with the highest score.



# Scoring models and parsing algorithms

Distinguish two aspects:

- **Scoring model:**

How do we want to score dependency trees?

- **Parsing algorithm:**

How do we compute a highest-scoring dependency tree under the given scoring model?



# The arc-factored model

- Split the dependency tree  $t$  into **parts**  $p_1, \dots, p_n$ , score each of the parts individually, and combine the score into a simple sum.

$$\text{score}(t) = \text{score}(p_1) + \dots + \text{score}(p_n)$$

- The simplest scoring model is the **arc-factored model**, where the scored parts are the arcs of the tree.



# Examples of classic features

- ‘The head is a verb.’
- ‘The dependent is a noun.’
- ‘The head is a verb  
*and* the dependent is a noun.’
- ‘The head is a verb  
*and* the predecessor of the head is a pronoun.’
- ‘The arc goes from left to right.’
- ‘The arc has length 2.’



# Training using structured prediction

- Take a sentence  $w$  and a gold-standard dependency tree  $g$  for  $w$ .
- Compute the highest-scoring dependency tree under the current weights; call it  $p$ .
- Increase the weights of all features that are in  $g$  but not in  $p$ .
- Decrease the weights of all features that are in  $p$  but not in  $g$ .





# Training using structured prediction

- Training involves repeatedly parsing (treebank) sentences and refining the weights.
- Hence, training presupposes an efficient parsing algorithm.



# Higher order models

- The arc-factored model is a first-order model, because scored subgraphs consist of a single arc.
- An  $n$ th-order model scores subgraphs consisting of (at most)  $n$  arcs.
- Second-order: siblings, grand-parents
- Third-order: tri-siblings, grand-siblings
- Higher-order models capture more linguistic structure and give higher parsing accuracy, but are less efficient



# Parsing algorithms

- Projective parsing
  - Inspired by the CKY algorithm
    - Collins' algorithm
    - Eisner's algorithm
- Non-projective parsing:
  - Minimum spanning tree (MST) algorithms
    - e.g. Chu-Liu-Edmunds algorithm (CLE)



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# Collins' algorithm



# Collins' algorithm

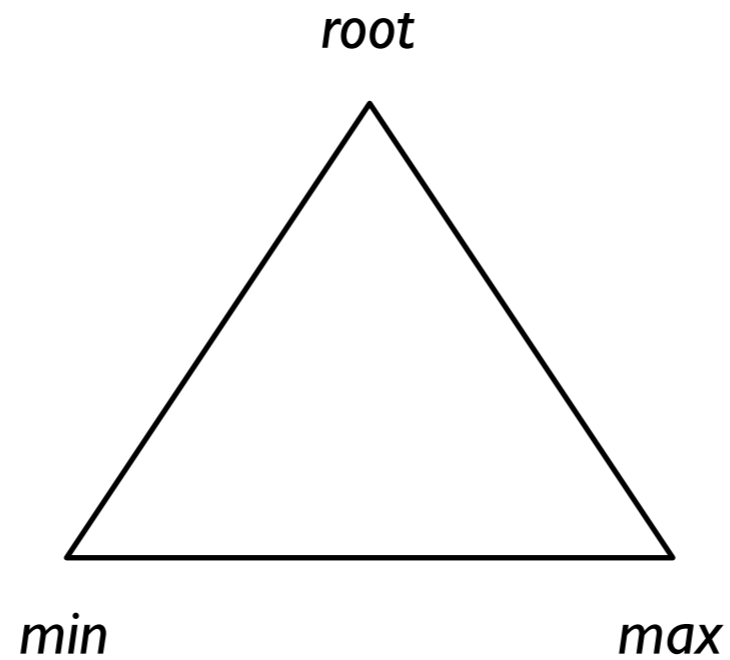
- Collins' algorithm is a simple algorithm for computing the highest-scoring dependency tree under an arc-factored scoring model.
- It can be understood as an extension of the CKY algorithm to dependency parsing.
- Like the CKY algorithm, it can be characterized as a bottom-up algorithm based on dynamic programming.



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Collins' algorithm

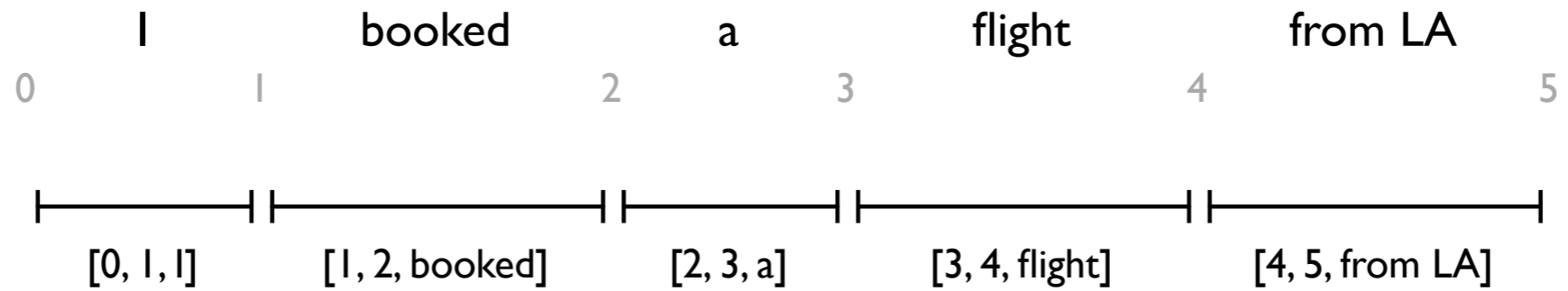
# Signatures, Collins'



**[*min, max, root*]**

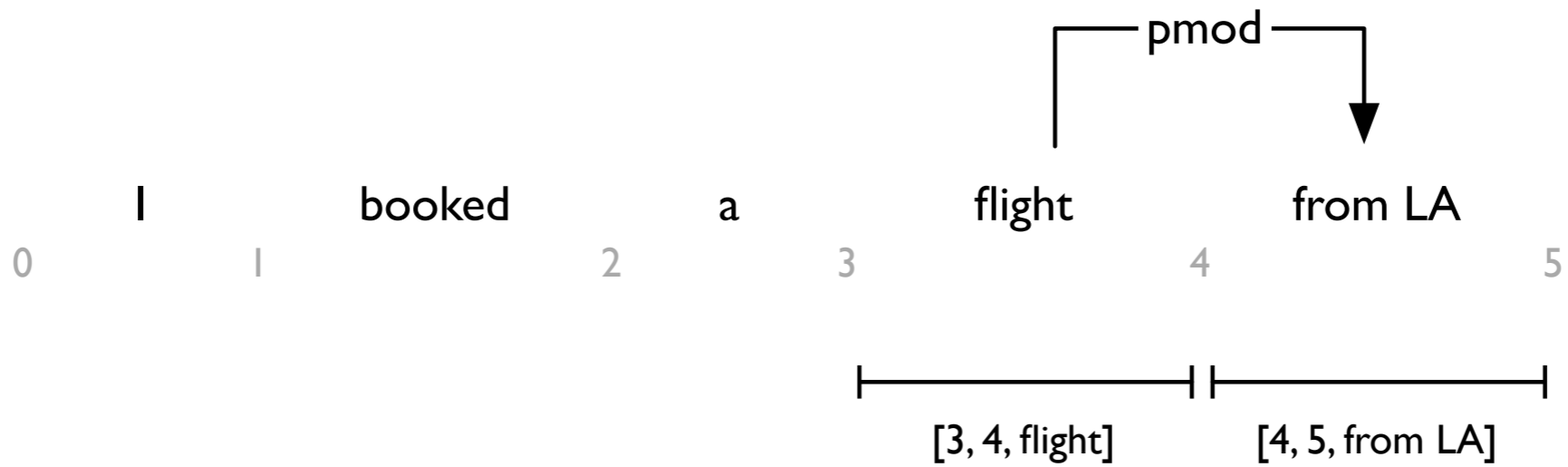


# Initialization





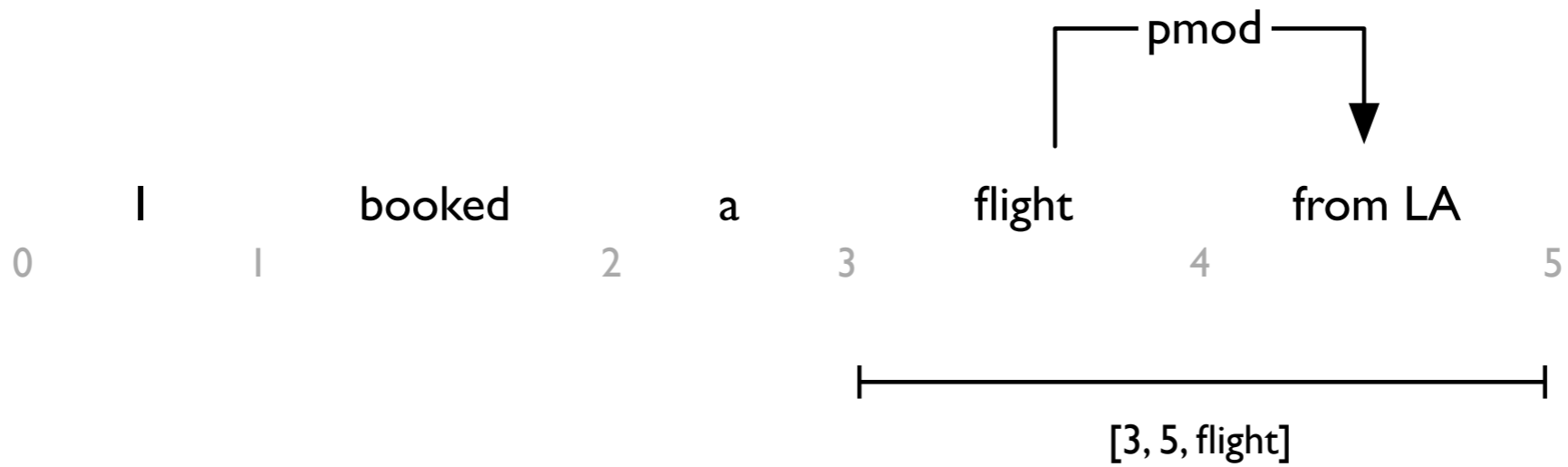
# Adding a left-to-right arc







# Adding a left-to-right arc

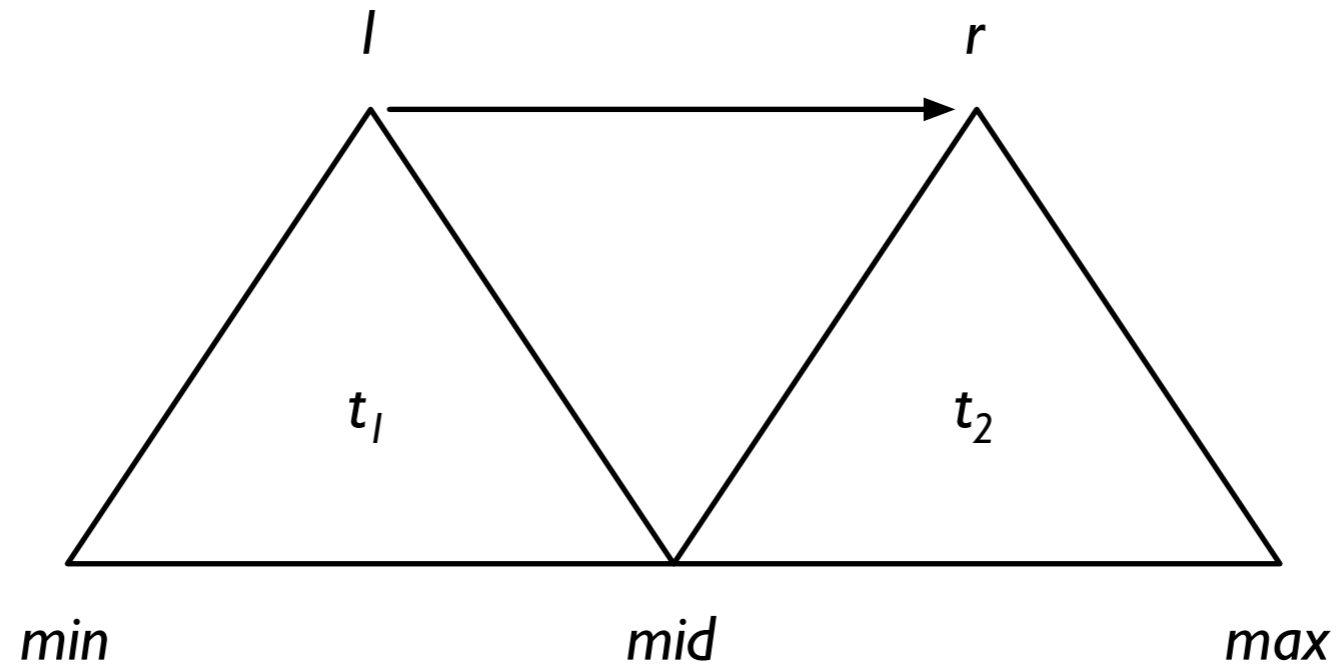




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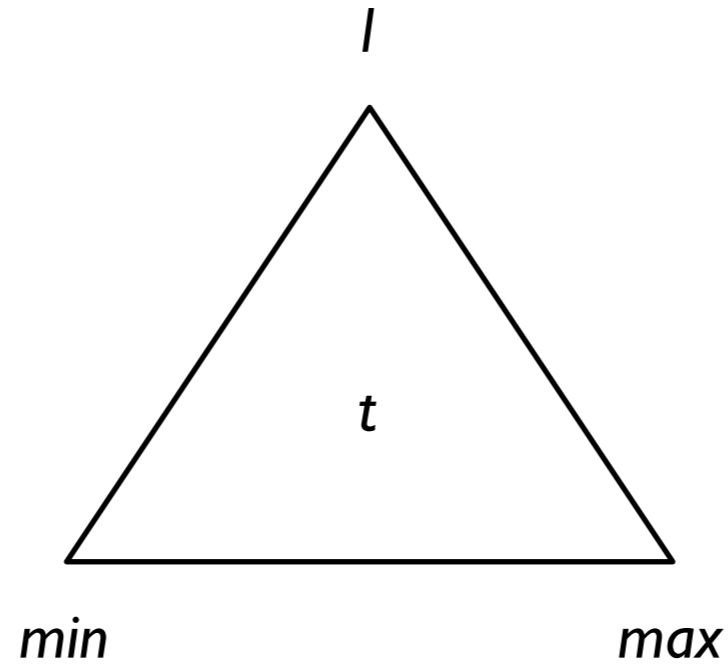
Collins' algorithm

# Adding a left-to-right arc





# Adding a left-to-right arc



$$\text{score}(t) = \text{score}(t_1) + \text{score}(t_2) + \text{score}(l \rightarrow r)$$



# Adding a left-to-right arc

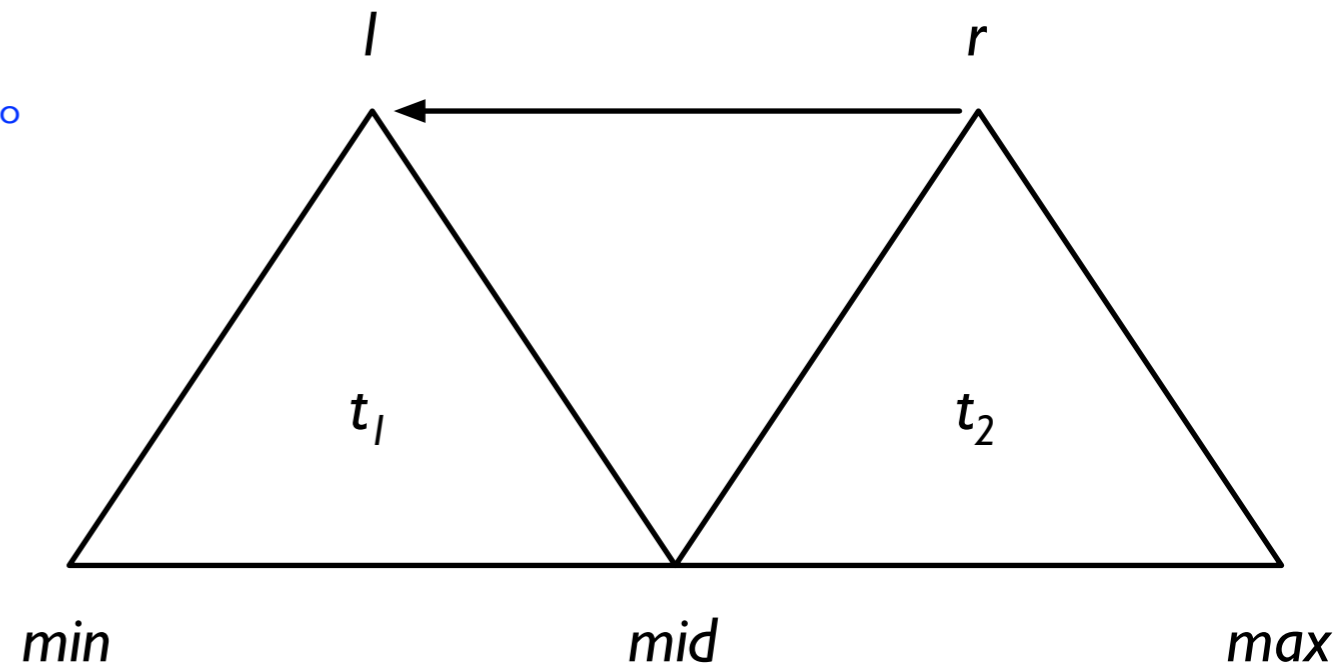
```
for each [min, max] with max - min > 1 do
  for each l from min to max - 2 do
    double best = score[min][max][l]
    for each r from l + 1 to max - 1 do
      for each mid from l + 1 to r do
        t1 = score[min][mid][l]
        t2 = score[mid][max][r]
        double current = t1 + t2 + score(l → r)
        if current > best then
          best = current
    score[min][max][l] = best
```



# Complexity analysis

- Runtime?
- Space?

```
for each [min, max] with max - min > 1 do  
  for each r from min + 1 to max - 1 do  
    double best = score[min][max][r]  
    for each l from min to r - 1 do  
      for each mid from l + 1 to r do  
        t1 = score[min][mid][l]  
        t2 = score[mid][max][r]  
        double current = t1 + t2 + score(r → l)  
        if current > best then  
          best = current  
    score[min][max][r] = best
```





# Complexity analysis

- **Space requirement:**  
 $O(|w|^3)$
- **Runtime requirement:**  
 $O(|w|^5)$



## Extension to the labeled case

- It is important to distinguish dependencies of different types between the same two words.

*Example:* subj, dobj

- For this reason, practical systems typically deal with **labeled arcs**.
- The question then arises how to extend Collins' algorithm to the labeled case.



## Smart approach

- Before parsing, compute a table that lists, for each head-dependent pair  $(h, d)$ , the label that maximizes the score of arcs  $h \rightarrow d$ .
  - This is guaranteed to be the arcs that could be used in a highest-scoring tree
- During parsing, simply look up the best label in the pre-computed table.
- This adds (not multiplies!) a factor of  $|L||w|^2$  to the overall runtime of the algorithm.



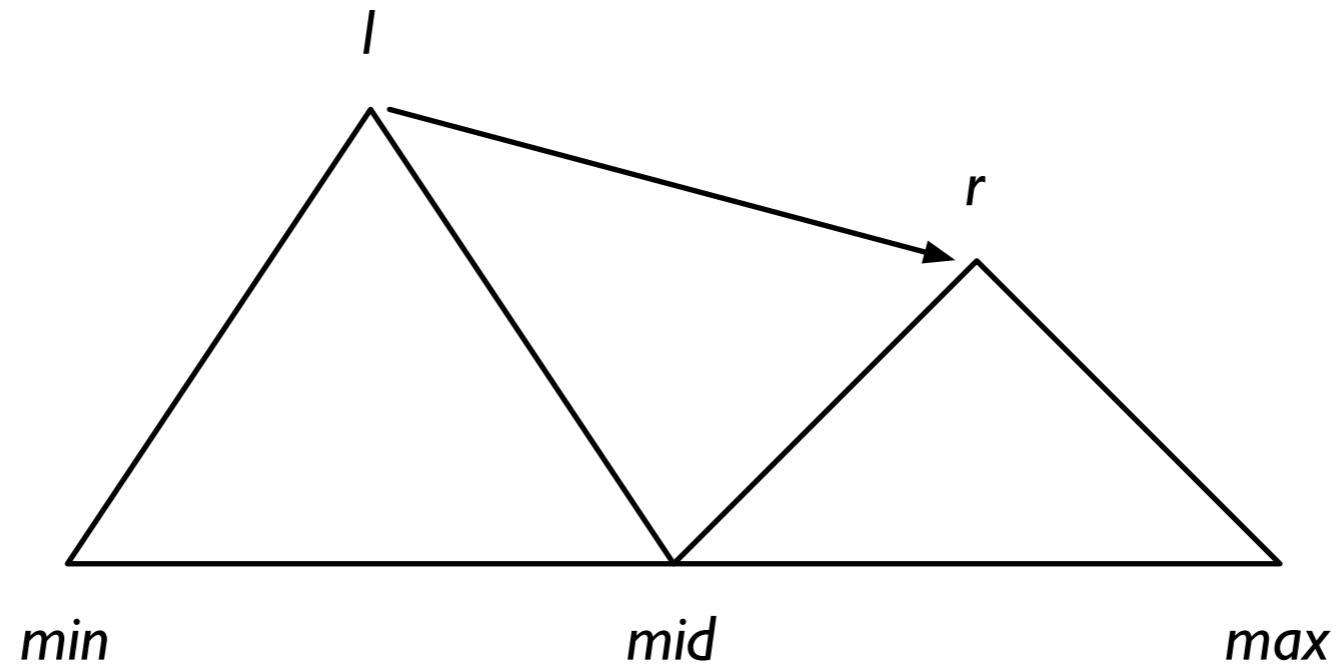


# Eisner's algorithm

- With its runtime of  $O(|w|^5)$ , Collins' algorithm may not be of much use in practice.
- With Eisner's algorithm we will be able to solve the same problem in  $O(|w|^3)$ .
  - Intuition: collect left and right dependents independently



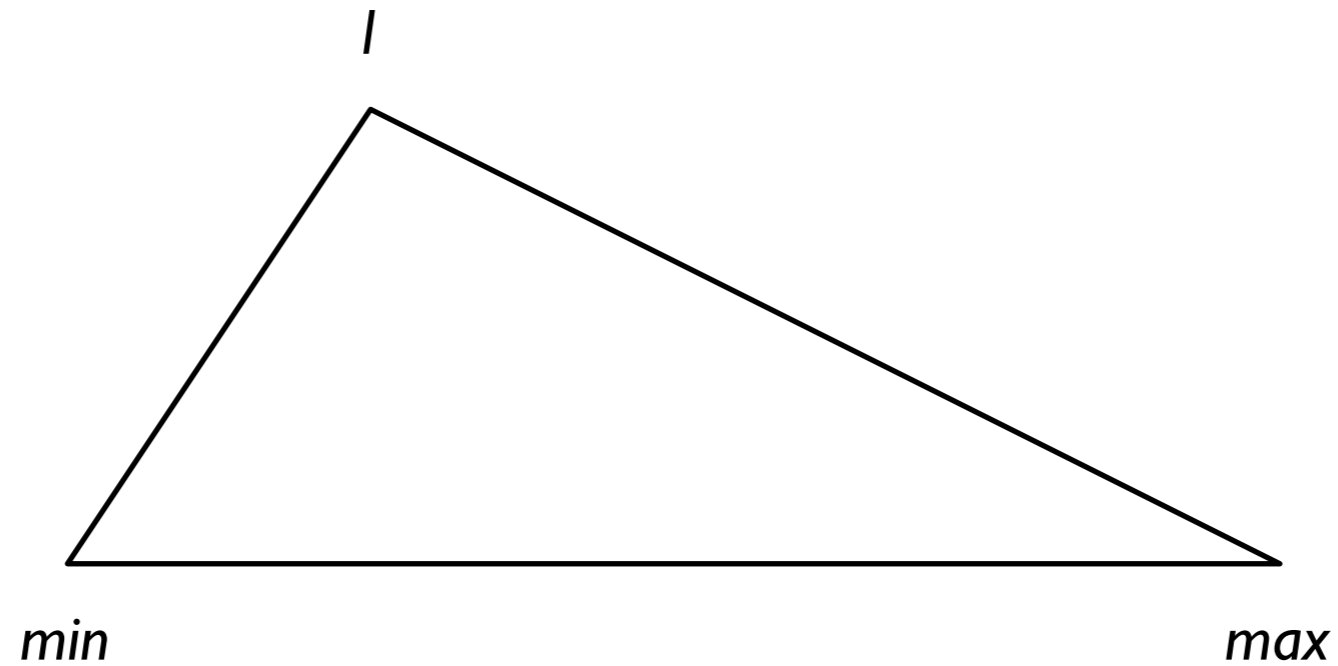
# Basic idea



In Collins' algorithm, adding a left-to-right arc is done in one single step, specified by 5 positions.



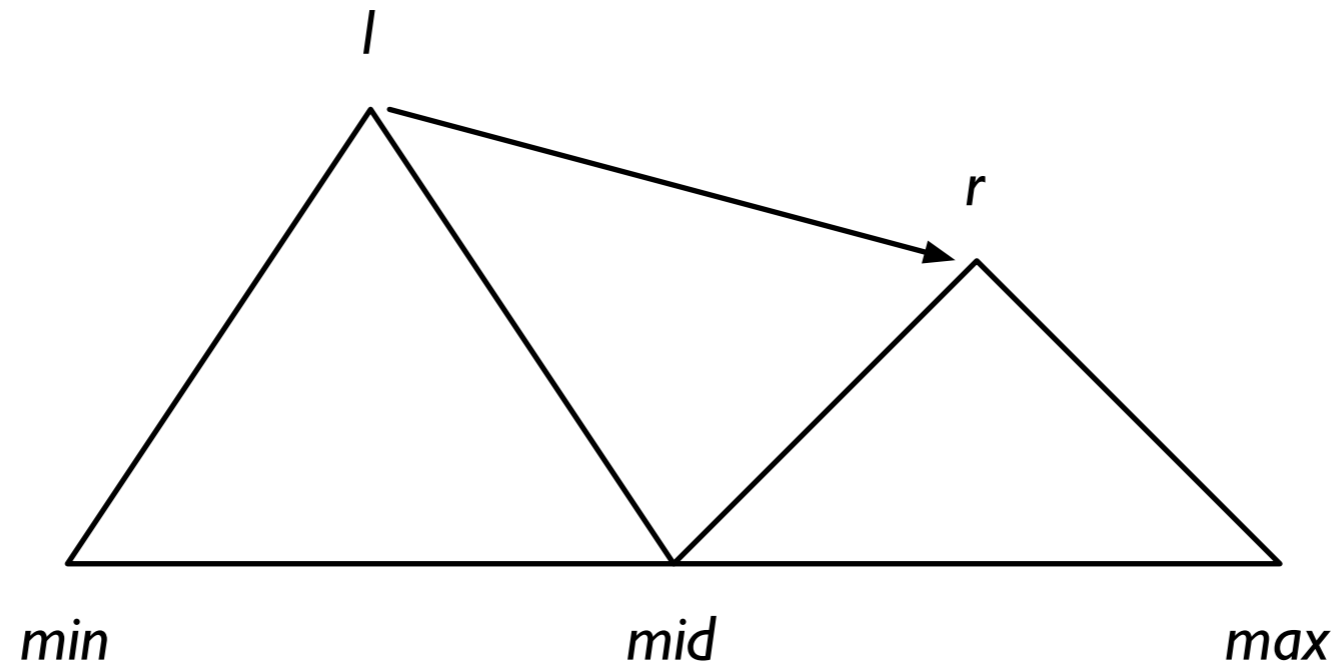
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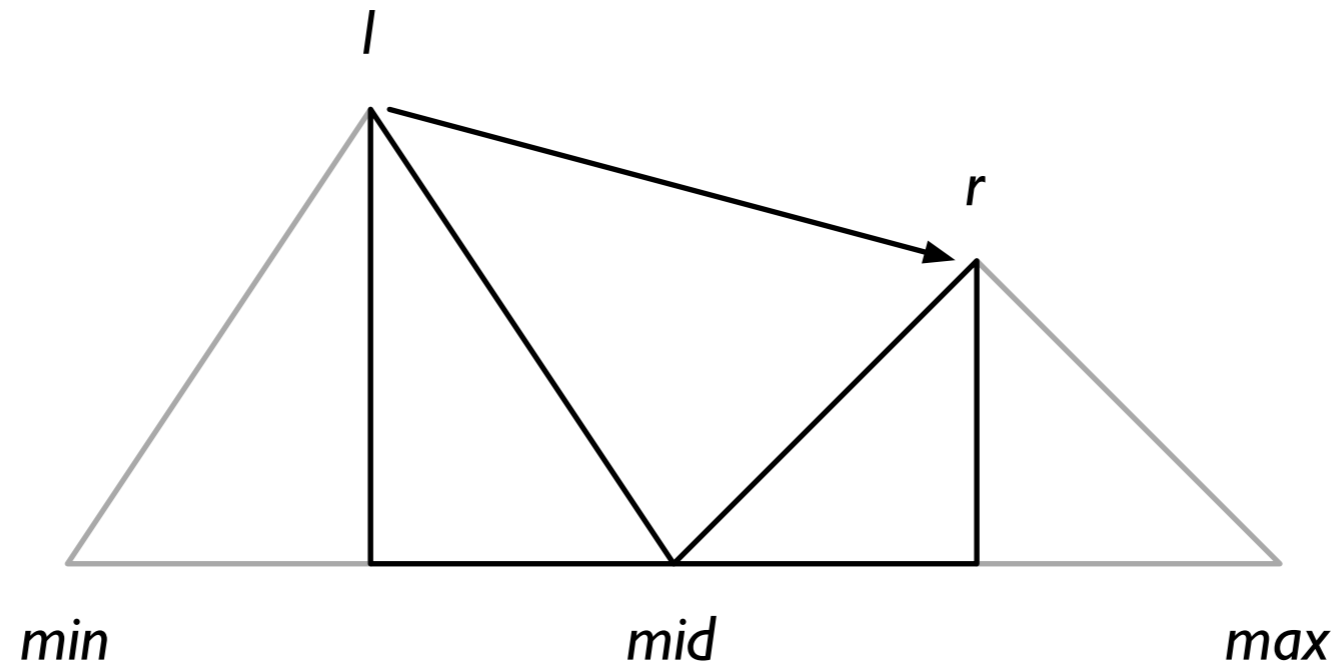
# Basic idea



In Eisner's algorithm, the same thing is done in three steps, each one specified by 3 positions.



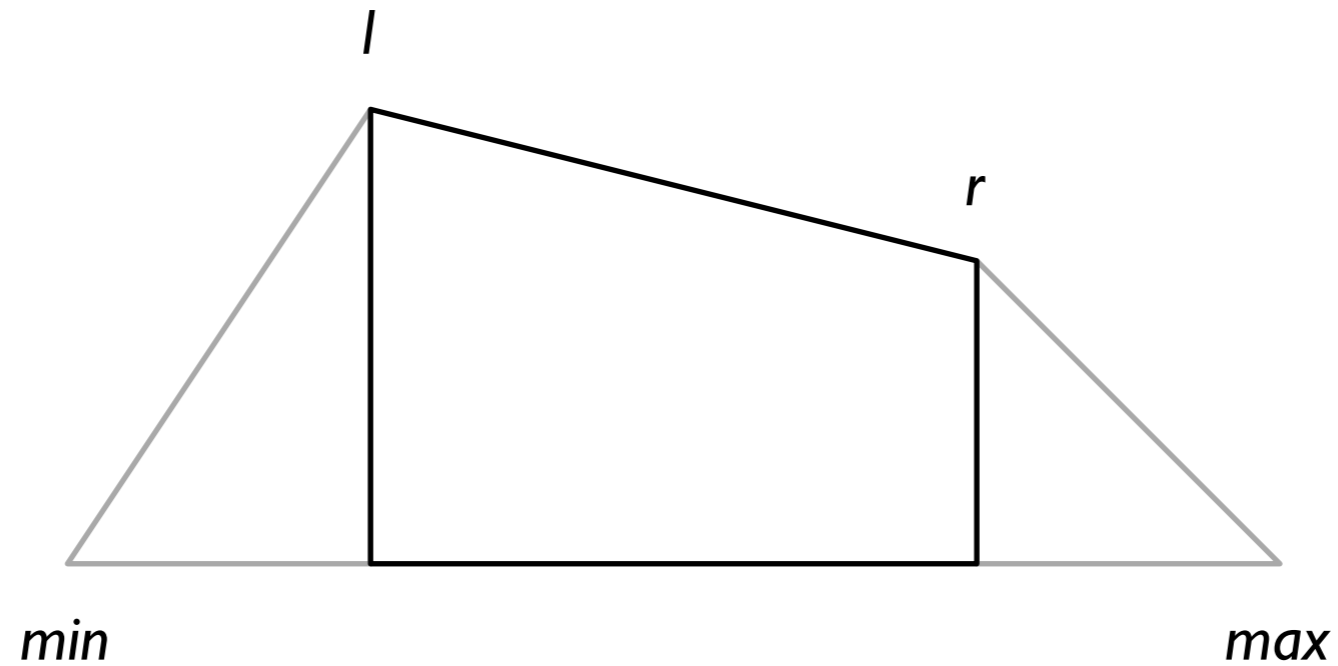
# Basic idea



In Eisner's algorithm, the same thing is done in three steps, each one specified by 3 positions.



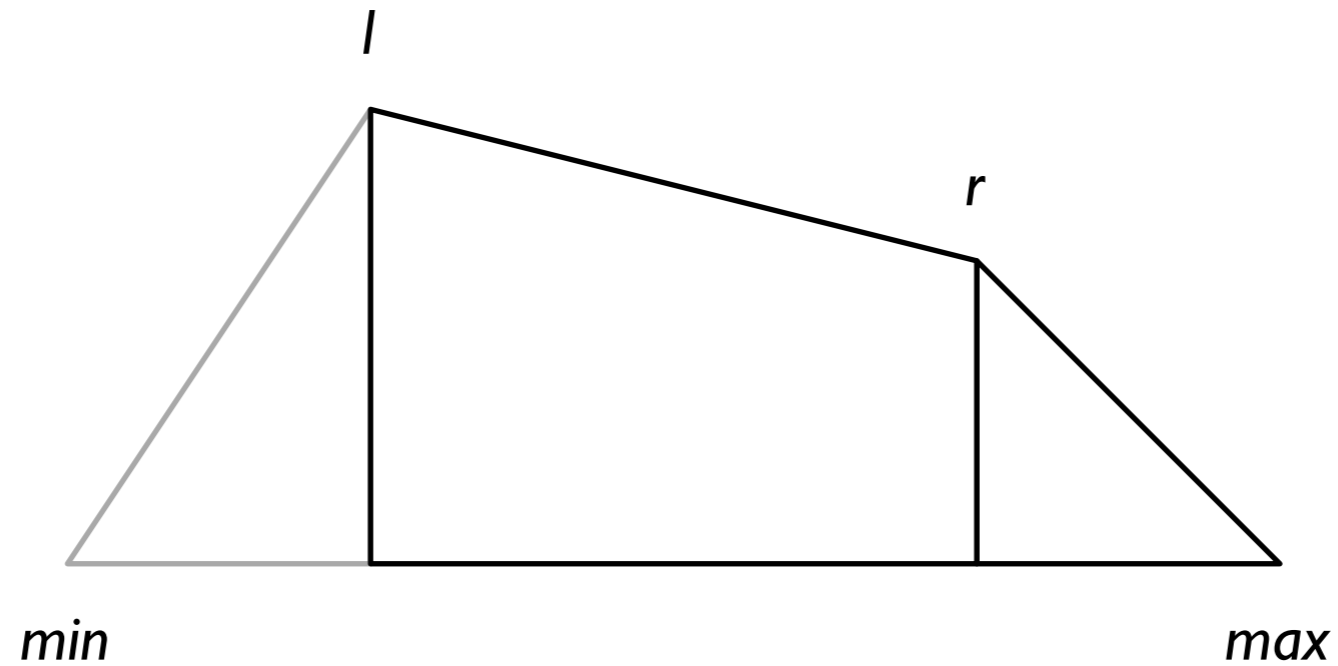
# Basic idea



In Eisner's algorithm, the same thing is done in three steps, each one specified by 3 positions.



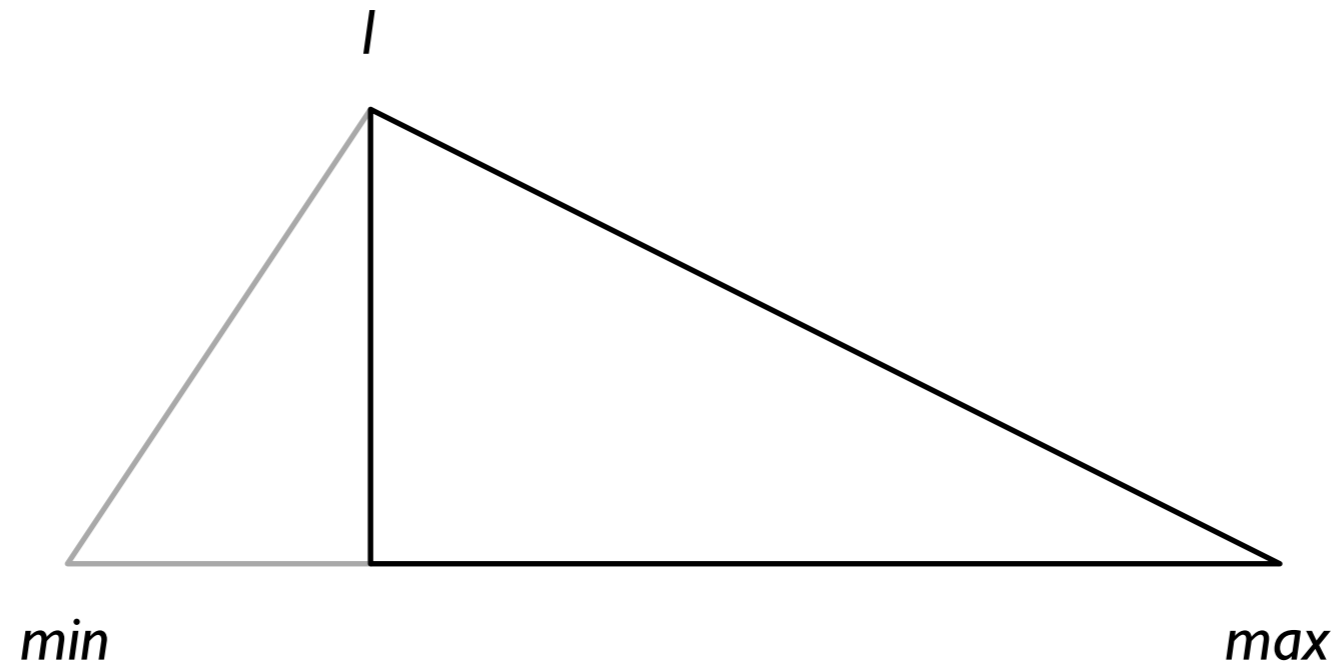
# Basic idea



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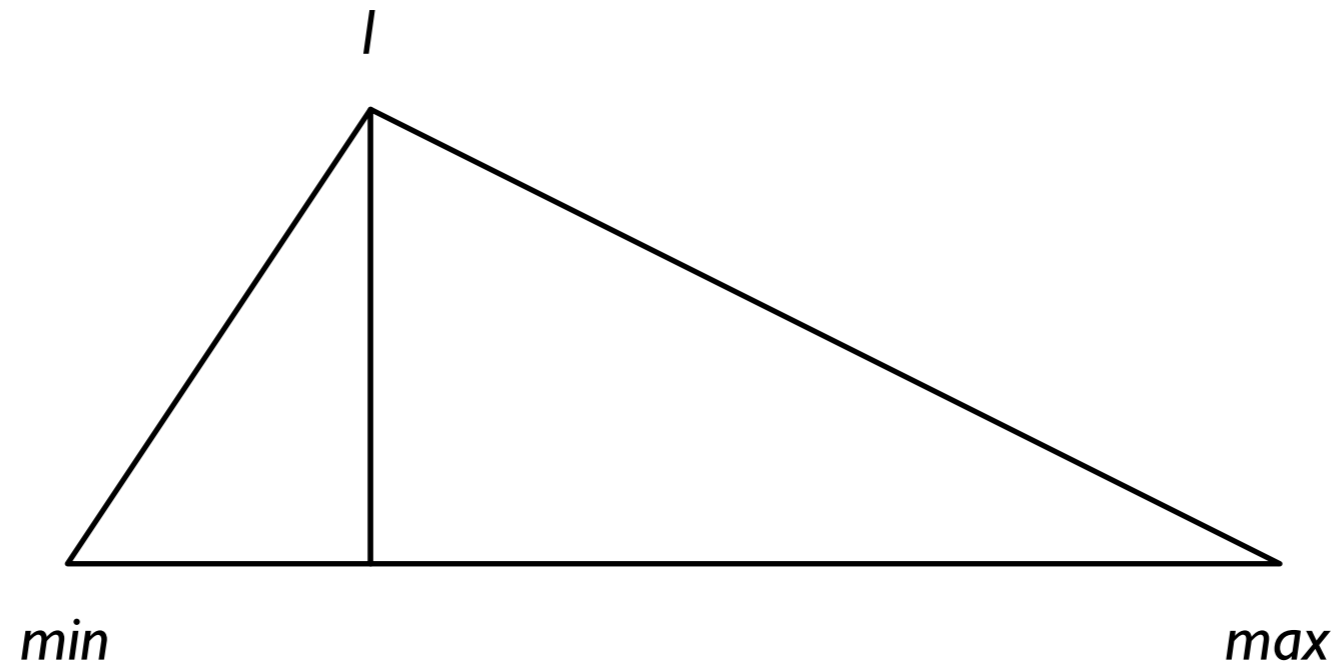




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Eisner's algorithm

# Basic idea



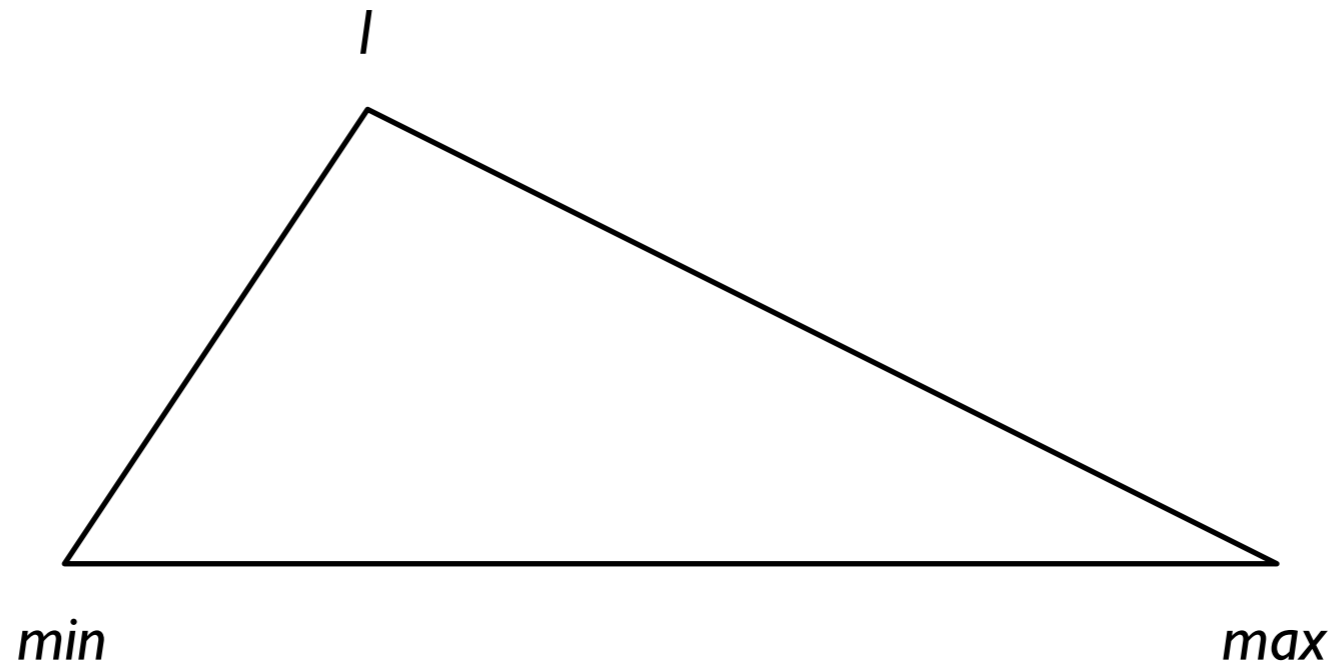
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Eisner's algorithm

# Basic idea



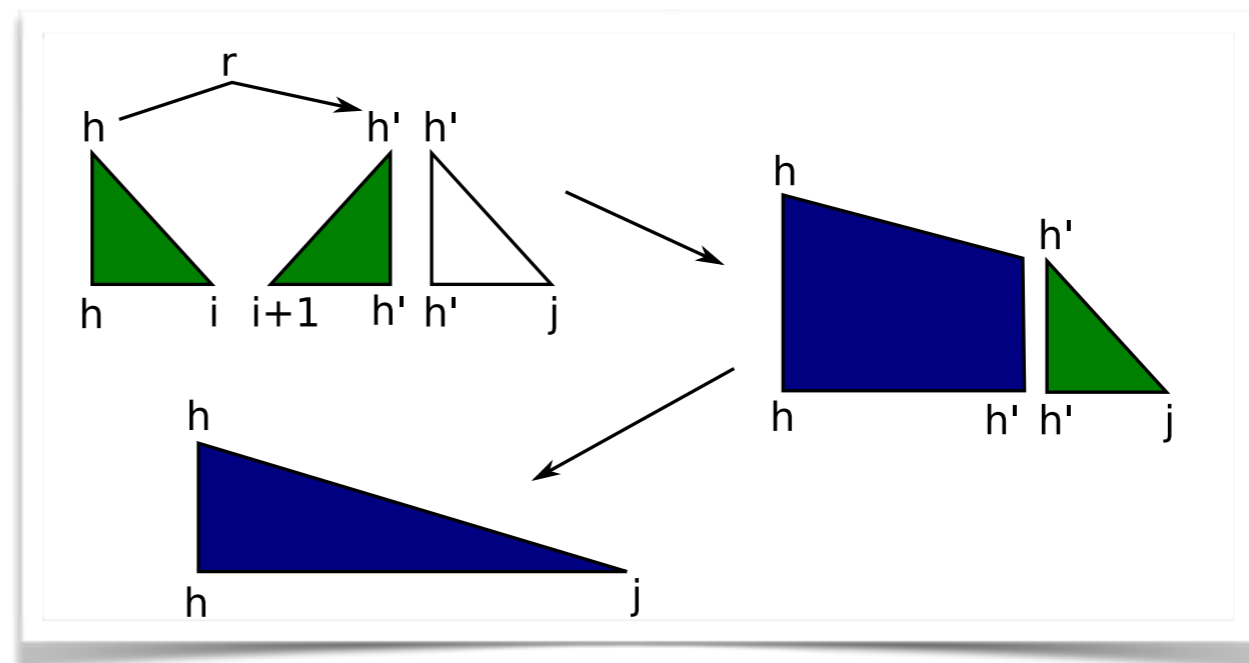
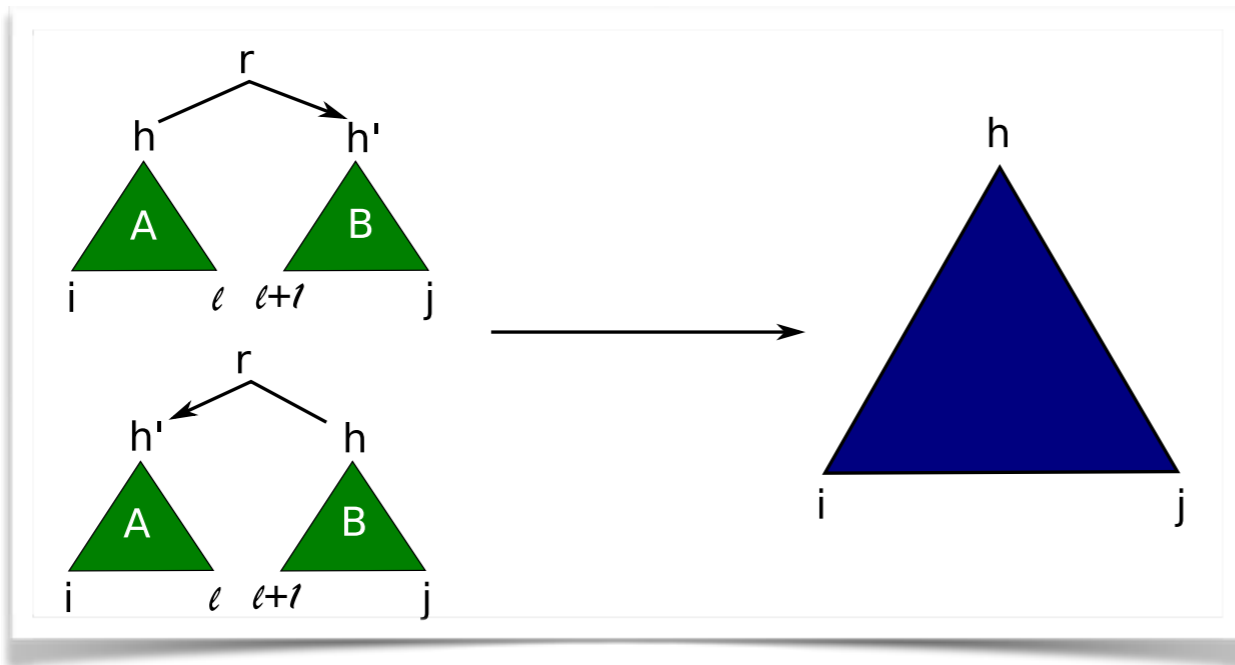
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Eisner's algorithm

# Comparison

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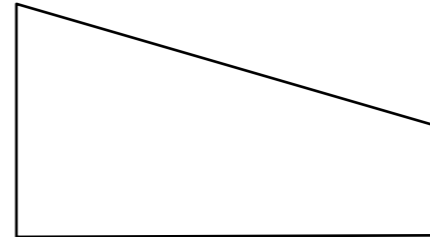
# Dynamic programming tables

- Collins':
  - [min,max,head]
- Eisner's
  - [min,max,head-side,complete]
    - head-side (binary): is head to the left or right?
    - complete (binary:) is the non-head side still looking for dependents?

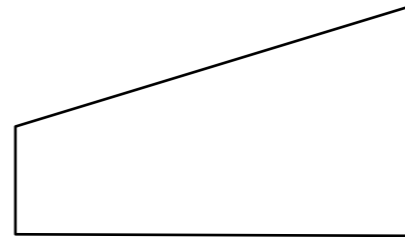


# Graphic representation

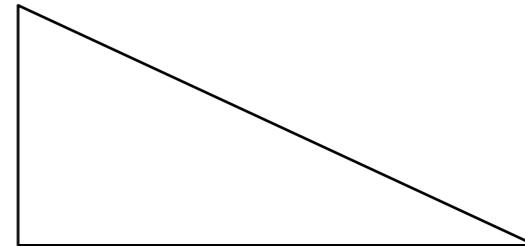
- [min,max,left,yes]



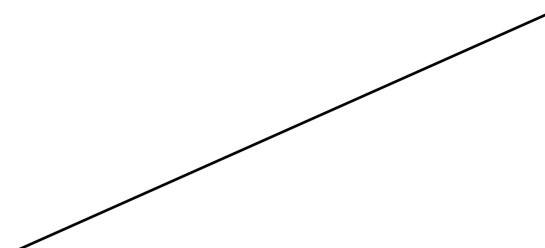
- [min,max,right,yes]



- [min,max,left,no]



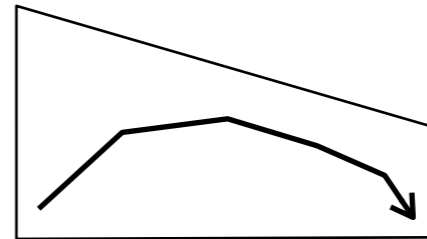
- [min,max,right,no]



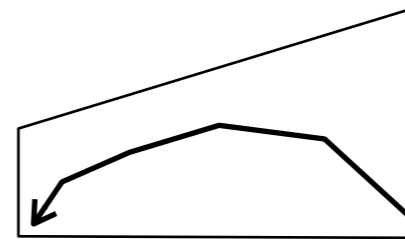


# Graphic representation

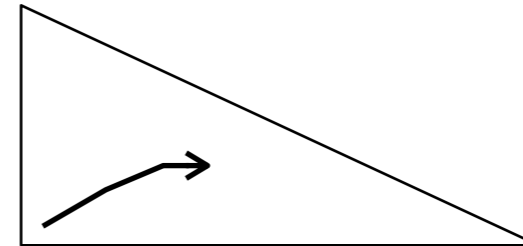
- [min,max,left,yes]



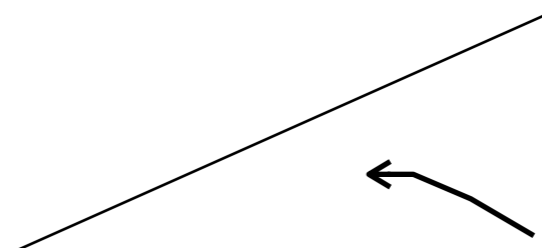
- [min,max,right,yes]



- [min,max,left,no]

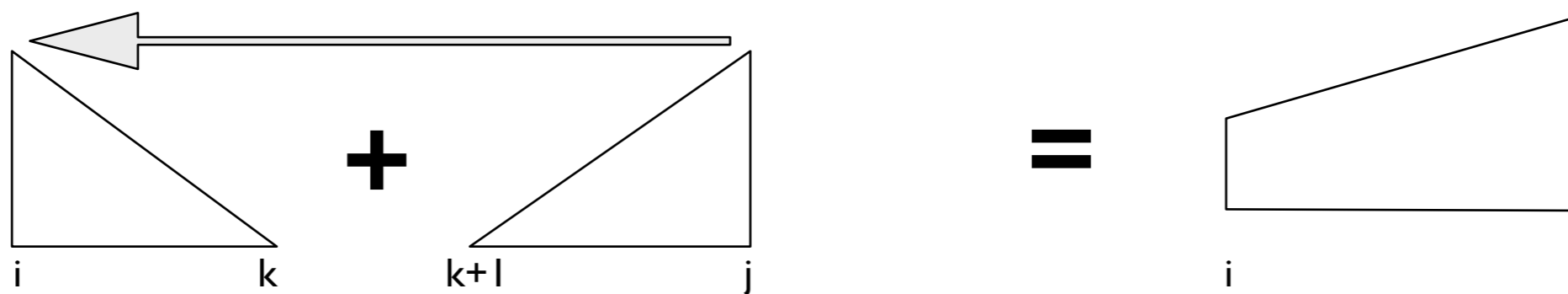
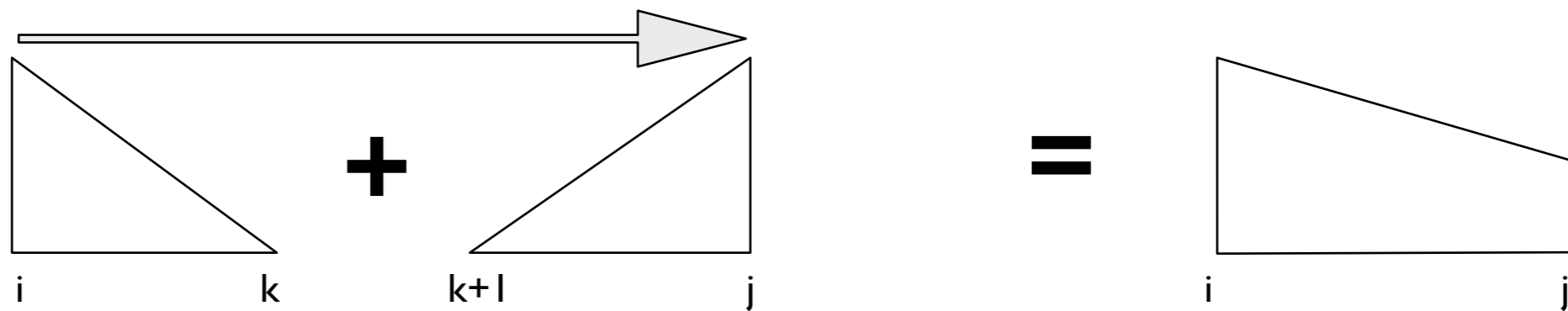
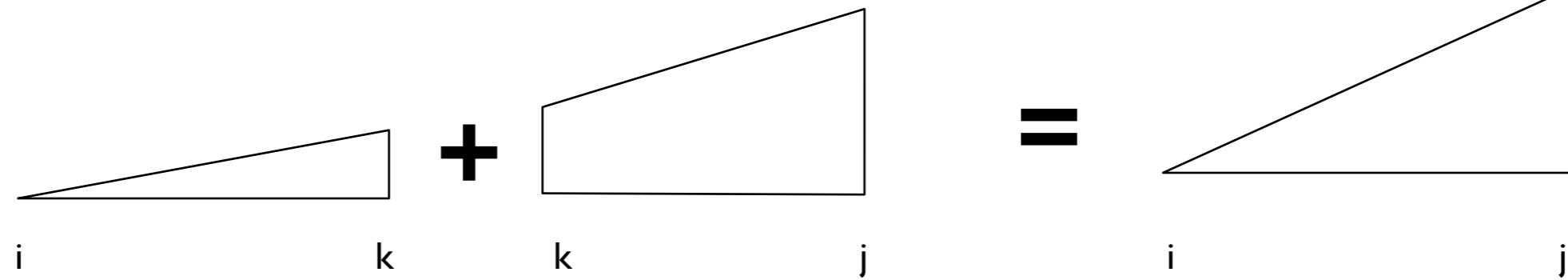
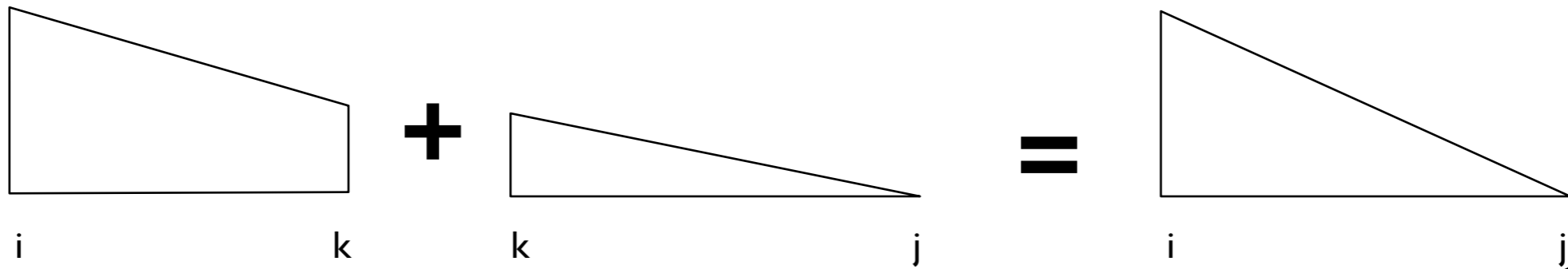


- [min,max,right,no]





# Possible operations





## Pseudo code

```
for each i from 0 to n and all d,c do
```

```
    C[i][i][d][c] = 0.0
```

```
for each m from 1 to n do
```

```
    for each i from 0 to n-m do
```

```
        j = i+m
```

```
        C[i][j][←][1] = maxi≤q<j(C[i][q][→][0] + C[q+1][j][←][0]+score(wj,wi))
```

```
        C[i][j][→][1] = maxi≤q<j(C[i][q][→][0] + C[q+1][j][←][0]+score(wi,wj))
```

```
        C[i][j][←][0] = maxi≤q<j(C[i][q][←][0] + C[q][j][←][1])
```

```
        C[i][j][→][0] = maxi≤q<j(C[i][q][→][1] + C[q][j][→][0])
```

```
return [0][n][→][0]
```





# Summary

- Eisner's algorithm is an improvement over Collin's algorithm that runs in time  $O(|w|^3)$ .
- The same scoring model can be used.
- The same technique for extending the parser to labeled parsing can be used, adding  $O(|L||w|^2)$  to the run time.
- Eisner's algorithm is the basis of current arc-factored dependency parsers.



# Minimum-spanning tree parsing

- Based on graph algorithms to find the minimum spanning tree
  - Often: Chu-Liu-Edmonds algorithm (CLU)
- Directly produces non-projective trees
- First suggested in the MSTparser
- One of the most popular algorithms today



# Minimum-spanning tree parsing

- **Intuition:**
- Score all word pairs in both directions
- Create a fully connected graph with these scores
- Remove all edges going into ROOT
- For each node, greedily keep only the highest-scoring incoming arc
  - If this produces a tree: done!
  - Otherwise: handle each cycle in the graph



# Evaluation of dependency parsers

- **labelled attachment score (LAS):**  
percentage of correct arcs,  
relative to the gold standard
- **labelled exact match (LEM):**  
percentage of correct dependency trees,  
relative to the gold standard
- **unlabelled attachment score/exact match (UAS/  
UEM):**  
the same, but ignoring arc labels



# Accuracy vs precision/recall

- Attachment score is an accuracy score
- For phrase-structure parsing we reported precision and recall
- Why is that not done for dependency parsing?



# Coming up

- Monday, Feb 20: guest lecture, Paola Merlo, 2-K1023
- Wednesday, Feb 22, Lecture:
  - Transition-based parsing (watch videos first)
- Sign up for a project and hand in a proposal in Studium (DL: February 27)
- Literature seminar 2, March 2
- Do assignment 2, literature review (DL: March 6)
- Start looking at the dependency assignment (DL: March 13)
  - Supervision: Feb 27 and March 8